Lecture 2: Atomic Structure and the Mass-Energy Equivalence

CBE 30235: Introduction to Nuclear Engineering — D. T. Leighton

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1 Classification of Elementary Particles (Lamarsh §2.1)

To understand the energy released in nuclear reactions, we must first categorize the subatomic "players." In modern physics, we classify particles not just by their mass, but by their **quantum spin** and how they interact with fundamental forces.

1.1 Fermions: The Building Blocks

Fermions are particles with half-integer spin ($\frac{1}{2}, \frac{3}{2}, \dots$). For nuclear engineers, the most important property of fermions is that they obey the **Pauli Exclusion Principle**: *no two identical fermions can occupy the same quantum state simultaneously.* This principle prevents the nucleus from collapsing into a single point and dictates the "shell" structure of the nucleus, analogous to electron shells in chemistry.

- **Quarks:** The fundamental constituents of nuclear matter. They possess fractional electric charges ($+\frac{2}{3}e$ or $-\frac{1}{3}e$). Due to *confinement*, quarks are never found in isolation; they are always bound within larger particles (Hadrons).
- **Leptons:** Particles that do not experience the "Strong" nuclear force.
  - **Electron (e^-):** The carrier of electricity and architect of chemical bonds. In nuclear physics, we are primarily interested in electrons when they are ejected from the nucleus during **Beta (β^-) decay**.
  - **Neutrino (ν):** A nearly massless, neutral particle. Because it interacts so weakly with matter, it was only discovered by the "missing" energy observed in beta decay. In this course, we treat the neutrino as a necessary "bookkeeper" for energy and momentum conservation.

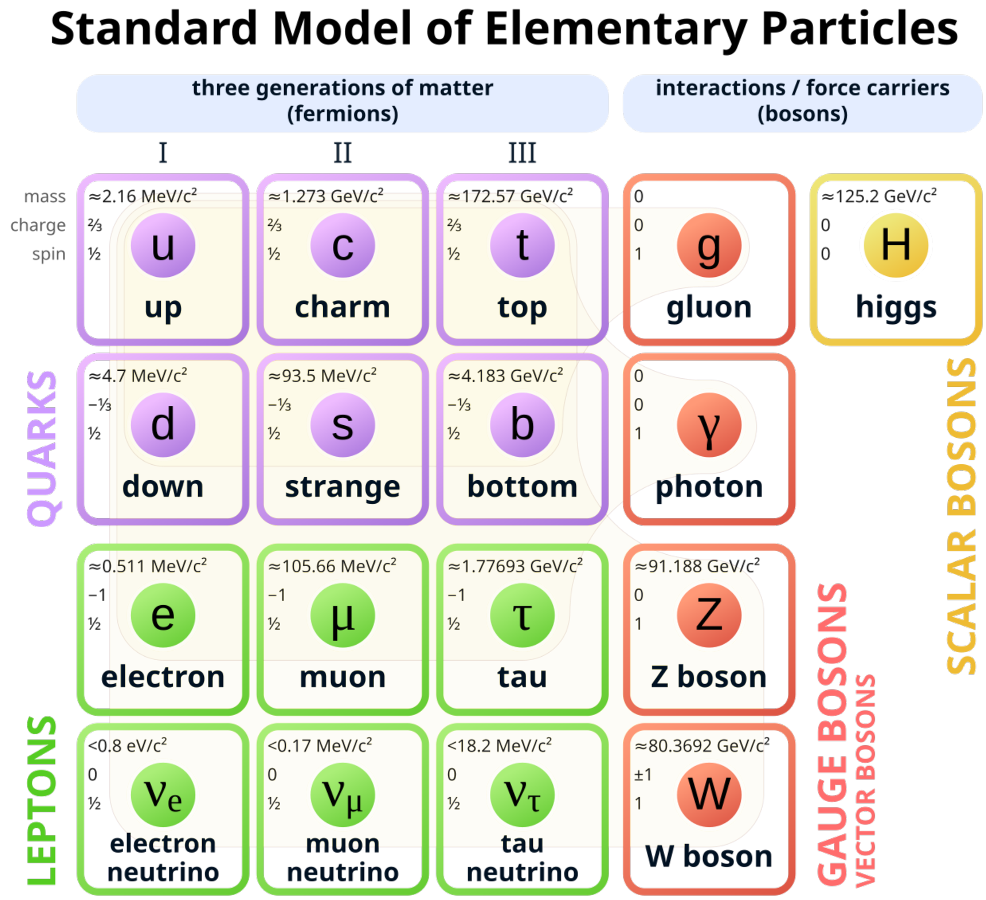

Figure 1: The Standard Model of Elementary Particles. Source: Particle Data Group.

1.2 Hadrons: Composite Particles

Hadrons are complex particles made of quarks held together by the Strong Force. We divide them into two categories:

- **Baryons (Fermions):** Particles made of **three quarks**. Protons and neutrons (collectively called **nucleons**) are the most stable baryons.
  - **Proton** (uud)**:** Two "up" quarks and one "down" quark. Net charge: $(+2/3 + 2/3 - 1/3) = +1e$.
  - **Neutron** (udd)**:** One "up" quark and two "down" quarks. Net charge: $(+2/3 - 1/3 - 1/3) = 0$. The neutron is slightly heavier than the proton, which is critical for the stability of atoms.
- **Mesons (Bosons):** Particles made of **one quark and one antiquark**. While unstable, mesons (specifically *pions*) are the particles "exchanged" between nucleons to create the residual strong force that binds the nucleus together.

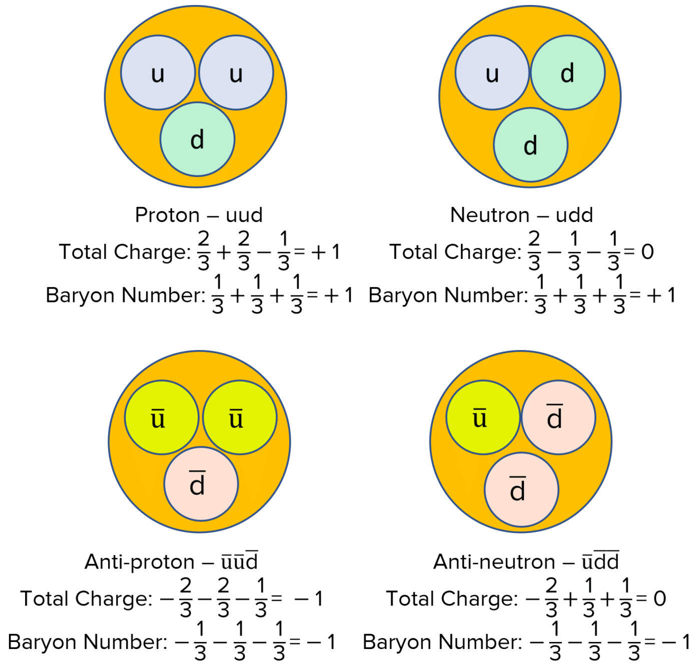

Figure 2: Quark structure of the Proton (uud) and Neutron (udd). Source: mmerevise.co.uk

1.3 Bosons: The Force Carriers

Bosons have integer spin $(0, 1, 2, \dots)$. Unlike fermions, multiple bosons *can* occupy the exact same quantum state. They act as the messengers of the fundamental forces.

- **Photon (γ):** The carrier of the **Electromagnetic Force**. In the nucleus, photons mediate the **Coulomb Repulsion** between protons. High-energy photons emitted from the nucleus are termed **Gamma Radiation**.

- **Gluon (g):** The carrier of the **Strong Force**. Gluons "glue" quarks together to form nucleons. On a larger scale, the residual effect of this force is what overcomes Coulomb repulsion to hold the nucleus together.

  Engineering Context: Think of the nucleus as a high-pressure vessel. The **Strong Force** provides the "tensile strength" of the vessel walls, while the **Coulomb Force** acts as the internal pressure. If the number of protons grows too large, the internal pressure can exceed the tensile strength, leading to instability or fission.

A Note on Mass Origins: Interestingly, the rest masses of the constituent quarks account for only $\sim$1% of a nucleon's mass. The remaining 99% arises from the kinetic energy of the quarks and the energy of the gluon field holding them together. At this sub-nuclear level, energy literally **is** mass.

Recommended Resources for Section 1

- **Textbook:** Lamarsh & Baratta, *Introduction to Nuclear Engineering*, Chapter 2, Section 2.1.

- **Interactive Guide:** The Particle Adventure (LBNL) - See the "Standard Model" and "What is a Quark?" sections.
- **Visualizing the Strong Force:** HyperPhysics: Fundamental Forces.

2 Atomic and Nuclear States (Lamarsh §2.2)

Nuclei are characterized by:

- **Z**: Atomic number (protons).
- **N**: Neutron number.
- **A**: Mass number ($Z + N$).

Isotopes have the same Z, **isobars** have the same A, and **isotones** have the same N.

3 Nuclear Dimensions and Stability (Lamarsh §2.3-4)

While the atom is mostly empty space, the nucleus is a region of extreme density. To understand how nucleons pack together, we rely on experimental data from high-energy electron scattering, which reveals a remarkably consistent structure across the periodic table.

3.1 The Nuclear Radius

Experimental evidence shows that the nucleus does not have a "hard" boundary, but rather a density that drops off rapidly at the edge. We approximate the nucleus as a sphere with a radius R:

$$R \approx R_0 A^{1/3} \tag{1}$$

Where:

- $R_0 \approx 1.25 \times 10^{-13}$ cm (or 1.25 fm).
- A is the Mass Number (total nucleons).

Note: The $A^{1/3}$ relationship is a direct consequence of the fact that nucleons are nearly incompressible and pack together like marbles in a bag.

3.2 Derivation: Constant Nuclear Density

A striking feature of the nucleus is that the density of nuclear matter is nearly constant, regardless of the element. We can derive this using the radius formula:

1. **Volume (V):** Assuming a spherical geometry:

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

2. **Mass (m):** The mass of the nucleus is approximately $A \times m_u$, where m_u is the atomic mass unit.

3. **Density (ρ):**

$$\rho = \frac{\text{Mass}}{\text{Volume}} \approx \frac{A \cdot m_u}{\frac{4}{3}\pi R_0^3 A}$$

Notice that the **A terms cancel out**. This implies that all nuclei—from Hydrogen to Uranium—have the same density:

$$\rho_{nuc} \approx \frac{m_u}{\frac{4}{3}\pi R_0^3} \approx 2.3 \times 10^{14} \text{ g/cm}^3 \tag{2}$$

Physical Comparison: If a teaspoon of nuclear matter were brought to Earth, it would weigh approximately 1 billion tons. This extreme density is why nuclear reactions involve energy scales millions of times greater than chemical reactions.

3.3 The Cosmic Connection: Neutron Stars

The nuclear density derived above ($\approx 10^{14}$ g/cm^3) is not just a theoretical construct. It is the physical density of **Neutron Stars**.

- When a massive star ($> 8M_\odot$) exhausts its fuel, the core collapses until it reaches nuclear density.
- At this point, **Neutron Degeneracy Pressure** (a consequence of the Pauli Exclusion Principle for neutrons) halts the collapse.
- The result is essentially a macroscopic nucleus with a radius of $\sim$10 km and a really big bang (type II supernova) which blows off most of the star. This is the cause of the pulsars at the center of nebulae such as the Crab Nebula. If the remaining core mass exceeds $\sim$3 $M_\odot$ (the TOV [Tolman-Oppenheimer-Volkoff] limit), even this pressure fails, and the object collapses into a Black Hole.

4 Atomic Weight and Energy Units (Lamarsh §2.5)

To perform engineering calculations, we must move between mass and energy with high precision. In the nuclear world, the Joule is far too large a unit, and the kilogram is far too heavy.

4.1 The Atomic Mass Unit (u)

The **unified atomic mass unit (u)** is defined as exactly 1/12 the mass of a single neutral ^{12}C atom.

- This definition includes the mass of the 6 protons, 6 neutrons, and 6 electrons, as well as the binding energy holding them together.
- 1 u = 1.660539×10^{-24} g.

4.2 Energy Equivalence: $E = mc^2$

It is a common misconception that mass is "converted" into energy. In reality, **mass is a property of energy.** According to Einstein's relation, any system that possesses energy also possesses a corresponding mass, $m = E/c^2$.

In nuclear reactions, when we observe a "mass defect," we are actually measuring the energy that was liberated (usually as heat or radiation) when the nucleons became bound together. Because the strong force is so powerful, the amount of energy released is large enough that the resulting change in mass is measurable.

4.2.1 Key Units and Conversions

In nuclear engineering, we use the **electron-volt (eV)**, specifically the **MeV** (10^6 eV).

- **The Conversion to Joules:** 1 eV is the kinetic energy gained by an electron accelerating through 1 Volt.

$$1 \text{ eV} = 1.60218 \times 10^{-19} \text{ J} \implies \mathbf{1} \text{ MeV} = \mathbf{1.60218 \times 10^{-13}} \text{ J} \tag{3}$$

- **The Conversion from Mass to Energy:** Using $E = mc^2$ for a mass of 1 u:

$$1 \text{ u} \cdot c^2 \approx 931.5 \text{ MeV} \tag{4}$$

Calculation Note: When performing calculations, you can treat c^2 simply as a conversion factor with the value 931.5 MeV/u. If you are working in SI units (kg, m, s), use $c \approx 2.9979 \times 10^8$ m/s to get the result in Joules.

4.3 The "Neutral Atom" Convention

In this course, we almost always use **Neutral Atomic Masses** (M) rather than bare nuclear masses (m).

Why? Because it is nearly impossible to measure the mass of a "naked" nucleus in a lab without its electrons. Our data tables (like Lamarsh Appendix II) provide atomic masses.

Particle	**Symbol**	**Mass (u)**
Neutron	m_n	1.008665
Proton	m_p	1.007276
Electron	m_e	0.000548
Hydrogen-1 Atom	$M(^1H)$	**1.007825**

Table 1: Key masses for calculations. Note that $M(^1H) \approx m_p + m_e$.

Critical Rule: When calculating the mass of the "parts" of an atom, use the mass of **Hydrogen-1** ($M(^1H)$) to account for the protons and their associated electrons simultaneously. This ensures the electrons cancel out when you subtract the final atomic mass.

5 Exit Hook: The Path to Stability

As Z increases, Coulomb repulsion grows. To stay stable, heavy nuclei need more neutrons ($N/Z > 1$). Next, we will explore the **Chart of the Nuclides** (§2.8) and how **Binding Energy** (§2.11) governs nuclear reactions.